

A MASSLESS FIREWALL

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Several anomalies of black hole thermodynamics are resolved if one accepts hints from semi classical theory that in regions of strong gravity the energy density of the vacuum can become substantially negative. This leads to a picture of the horizon as a hot massless shell. As viewed in the local (Boulware) ground state, the apparent horizon is the repository of a large store of energy, and this energy is thermal. But, as a source of gravity, its mass is near zero. A simple (1+1)-dimensional model provides a concrete realization of this picture.

I. INTRODUCTION

Our current picture of black hole evaporation contains some curious anomalies. The Hawking spectrum is thermal, which suggests a very hot source when one takes account of the strong blueshift approaching the horizon. Yet the energy density and field stresses in no way reflect this, remaining finite and moderate right up to the horizon.

The disparity appears in sharpest form for a hypothetical black hole that has reached equilibrium with its own radiation in a box (described by the Hartle-Hawking state). The local temperature T then increases with depth according to Tolman's law

$$T = T_H / (-g_{00})^{\frac{1}{2}}, \quad (1)$$

valid for any system in thermal equilibrium (T_H is a constant, the Hawking temperature). The local temperature becomes infinite at the horizon. Yet again, the stress-energy shows no evidence of this.

Two other states commonly appear in these discussions. The Boulware state, empty at infinity, is the ground state of quantum fields around a cold star. The Unruh state, empty at past infinity and regular on the future horizon, is customarily taken as ground state for an evaporating black hole [1].

The Boulware energy density is negative, and diverges for a star nearing its gravitational radius. Such negative energies must be considered real. An armchair experiment treated by Anderson [2] nicely illustrates this. In (1+1)-dimensions, a quantum mirror which is changing its acceleration emits radiation. Anderson considers a box with reflecting walls being slowly lowered in a gravitational field. The walls will radiate into the box energies of opposite sign but slightly out of balance because of the different accelerations of floor and ceiling. The result is that the box gradually fills with negative energy as it is lowered. The calculation shows that the energy density inside exactly matches the ambient Boulware density. Thus the negative energy has operational significance.

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While energy densities can become negative locally, the total mass-energy of the system remains fixed at its asymptotic value, and positive.

Consider now a star, with quantum fields in the Boulware state, slowly shrinking toward its gravitational radius. The moment a horizon forms, Hawking radiation turns on, with strong effects on the near-horizon geometry. We accordingly switch our choice of ground state from Boulware to Unruh. This is quite reasonable. The division into “ground state” and “excitations” is arbitrary, and, to keep the perturbations small, it is convenient to recalibrate so that the effects of the Hawking excitations becomes part of the background. However, this recalibration glosses over the fact that the difference between the old and new ground-state state energies is *thermal*, and thermality is not just a gauge effect. The excitations on the right-hand side of the semi-classical Einstein equations have been transferred to the left in geometrized form. Mathematically, the procedure is innocuous. But the thermal origin of the excitations has been obscured, so physical intuition regarding thermal interactions is affected, sometimes in odd ways.

There is no deeper justification, other than convenience, for the abrupt change in our choice of ground state when a horizon forms.

We must then be prepared for the (retrospectively obvious) consequence that, in this new (Unruh) gauge, *the large thermal energies near the horizon have only small gravitational and inertial effects*, and contribute little to the stress-energy on the right-hand side of Einstein’s equations. Near the horizon we have heat without mass.

This unveils a picture of the horizon as a nearly massless shell which nevertheless contains a large store of thermal energy—a “firewall”.

Remarkably, such a possibility was adumbrated recently on quite different grounds by Almheiri, Marolf, Polchinski and Sully [3]. They advocated a firewall as the “most conservative” solution to a paradox that arises when one examines correlations of late modes of Hawking radiation with their internal partners and with the early modes.

The firewall has earlier antecedents. ’t Hooft’s “brick wall” model [4, 5] for black hole entropy was a proposal to explain the Bekenstein-Hawking area law by actually localizing the entropy on the horizon. It now emerges that the brick wall is a firewall. Even earlier, there are connections with the “membrane paradigm” of Thorne and associates [6].

II. REDUCED SPHERICAL EINSTEIN THEORY

For a spherical metric in the general form

$$ds^2 = g_{ab} dx^a dx^b + r^2(x^a) d\Omega^2 \quad (2)$$

($a, b = 0, 1$), the 4-dimensional Einstein-Hilbert Lagrangian reduces to

$$L = \frac{1}{4} r^2 R + \frac{1}{2} (\nabla r)^2 + L_{\text{mat}}, \quad (3)$$

where R is the 2-dimensional curvature scalar.

We shall adopt this same Lagrangian to define a “spherical” Einstein theory in (1+1) dimensions, with $r(x^a)$ now an auxiliary scalar (“dilaton”) field.

Define the scalar fields f, m by

$$f = (\nabla r)^2 = 1 - 2m/r \quad (4)$$

Then (3) leads to the field equations [7]

$$\partial_a m = (T_a^b - \delta_a^b T_d^d) \partial_b r \quad (5)$$

$$r_{;ab} = \left(\square r - \frac{m}{r^2} \right) g_{ab} - \frac{1}{r} T_{ab}, \quad (6)$$

which imply the conservation laws

$$T_{a;b}^b = 0. \quad (7)$$

For access to the future horizon it is useful to work with an advanced time co-ordinate v . A general 2-metric then takes the form

$$ds^2 = 2e^\psi dv dr - f e^{2\psi} dv^2 \quad (8)$$

and the field equations (5), (6) become

$$m_r = -T_v^v, \quad m_v = T_v^r, \quad \psi_r = \frac{1}{r} T_{rr}. \quad (9)$$

(Subscripts on m and ψ indicate partial derivatives.)

The curvature scalar for metric (8) is

$$R = -2e^{-\psi} (\alpha + \psi_v)_r, \quad (10)$$

where $\alpha = \frac{1}{2} e^{-\psi} (f e^{2\psi})_r$ is related to the local surface gravity

$$\kappa \equiv (-g_{00})^{\frac{1}{2}} a = \alpha - \frac{1}{2} \partial_v (\ln f) \quad (11)$$

(=redshifted acceleration a of a static observer $r = \text{const.}$).

III. HAWKING RADIATION

As source T_a^b we consider a quantized massless scalar field propagating on this classical background. Expectation values of all components T_a^b can be obtained from the conservation laws and the 2-dimensional conformal trace anomaly [1]

$$T_a^a = \tilde{h} R, \quad \tilde{h} \equiv \hbar/24\pi. \quad (12)$$

By manipulation of the conservation laws and use of (12) one arrives at

$$\partial_r \{ f e^{2\psi} T_r^r + 2e^\psi T_v^r + \tilde{h} (\kappa^2 - 2\partial_v \kappa) \} = 0 \quad (13)$$

(details in the Appendix). This provides a convenient first integral of the conservation laws.

It is intuitively helpful to put physical flesh onto these equations by introducing an orthonormal basis (s^a, t^a) anchored to the curves $r = \text{const.}$. The unit tangent to these curves is labelled t^a in the exterior domain $f > 0$ where they are timelike, and s^a in the black hole interior $f < 0$. We also define the outgoing and ingoing lightlike vectors

$$l^a = t^a + s^a, \quad n^a = t^a - s^a. \quad (14)$$

The stress tensor T^{ab} for any state (but we have in mind the Unruh state) can then be decomposed into a (Boulware-like) fluid part and an outflux F :

$$T^{ab} = T_B^{ab} + F l^a l^b \quad (15)$$

where

$$T_B^{ab} = P_B s^a s^b + \rho_B t^a t^b \quad (16)$$

This is useful in the exterior, where the Unruh boundary conditions require $(P_B, \rho_B) \rightarrow 0$ at past infinity and we want to isolate the Hawking flux F .

Alternatively and equivalently, we can decompose into a (Hartle-Hawking-like) fluid part and an oppositely-signed influx:

$$T^{ab} = T_H^{ab} - F n^a n^b \quad (17)$$

with

$$P_H = P_B + 2F, \quad \rho_H = \rho_B + 2F. \quad (18)$$

This provides a clearer description of the interior by exposing the infalling flux of negative energy from the horizon that accompanies the Hawking outflux.

For the Unruh stress tensor in the decomposition (15), equation (13) reduces to

$$\partial_r \left[\tilde{h}(\kappa^2 - 2\partial_v \kappa) + |f|e^{2\psi} \begin{cases} P_B & (f > 0) \\ \rho_B & (f < 0) \end{cases} \right] = 0 \quad (19)$$

(the flux term F drops out). Hence the expression in square brackets can be an arbitrary function of v , which must however vanish to satisfy the Unruh boundary condition that the spacetime is empty at past lightlike infinity. Hence (19) integrates to

$$\begin{cases} P_B & (f > 0) \\ \rho_B & (f < 0) \end{cases} = -\tilde{h} \frac{\kappa^2 - 2\partial_v \kappa}{|f|e^{2\psi}}. \quad (20)$$

The singularity this produces in T_B^{ab} on the apparent horizon $f = 0$ (and hence threatens in the Unruh stresses) must be offset by the flux term in (15).

Equation (9) for ψ becomes

$$rf\psi_r = \rho_B + P_B + 4F \quad (21)$$

and the trace anomaly (12) gives

$$P_B - \rho_B = \tilde{\hbar}R. \quad (22)$$

From (21), one sees that regularity of ψ at $f = 0$ (hence regularity of R by (10)) requires the Hawking flux there to be given by

$$|f|e^{2\psi}F = \frac{1}{2}\tilde{\hbar}(\kappa_0^2 - 2\partial_v\kappa_0) \quad (f \rightarrow 0), \quad (23)$$

where $\kappa_0(v)$ is the surface gravity κ at the apparent horizon.

For slow evaporation, the expression on the left of (23) is nearly conserved, as can be seen from

$$\frac{d}{dr}(fe^\psi F) = \frac{4}{r}e^\psi F^2, \quad (24)$$

which follows from the conservation laws (d/dr is the rate of change along outgoing light rays). Thus the right-hand side is also nearly the Hawking flux at infinity:

$$F(r \rightarrow \infty) \approx \frac{1}{2}\tilde{\hbar}\kappa_0^2 \quad (25)$$

This is the standard expression for the Hawking flux in (1+1) dimensions.

We have thus arrived at a picture of the apparent horizon as the source of two opposing and oppositely-signed streams of thermal energy. The matter content of the horizon itself is most directly appreciated from T_H^{ab} in the decomposition (17), because here the added flux term is transverse, hence not blueshifted. Equations (18), (20) and (23) show that T_H^{ab} vanishes in highest order f^{-1} and hence is at most of order \hbar/m^2 when $f = 0$. The apparent horizon is virtually empty—but hot!

IV. SUMMARY AND CONCLUSIONS

With the recognition that the apparent horizon of a black hole, far from being inactive, is the seat of a very hot, yet massless shell—a “firewall” [3]—a number of curious features of black hole thermodynamics fall naturally into place. The reason why the hole’s thermal parameters are linked to its surface properties—entropy to surface area, temperature to surface gravity—is that they *are* just surface properties. Hawking radiation is just thermal radiation from a hot surface.

Most intriguingly, this may hold the key to a conservative solution of the puzzle of information loss. Dropping information into a black hole is, after all, not so different from throwing an encyclopaedia in the fire. All information can, in principle, be recovered by collecting and reassembling the smoke and ashes.

Appendix: Derivation of (13)

If the two conservation laws $T_{a;b}^b = 0$ are written out, and terms in $\partial_v f$ are eliminated between them, the result can be cast in the form

$$f e^\psi \partial_r T_r^r + 2 e^{-\psi} \partial_r (e^\psi T_v^r) + (T_r^r - \frac{1}{2} T_a^a) e^{-\psi} \partial_r (f e^{2\psi}) + \partial_v T_a^a = 0,$$

Inserting the trace anomaly (12) for T_a^a and the expression (10) for R into this equation leads, after some manipulations, to (13).

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